Improved Bounds for Novelty Games

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Abstract

We study the *novelty* game [LS24], where p non-communicating players each receive k integers from $[N]$ and output m integers from $[N]$. The players succeed if some player outputs an integer not among the pk inputs. We improve upper bounds on the minimal N admitting a guaranteed winning strategy. Focusing on oblivious strategies and the case $m = 1$, we develop a framework with six optimizations that sharpen the classical bound. For $(p, k, m) = (3, 2, 1)$ we reduce the best known bound from 1.71×10^8 to 193050. In general $(p, k, 1)$ settings, our analysis yields an upper bound of the form $\prod_{i=0}^{p-1} \binom{D(p,k)}{k^i}$, leading to an exponential reduction of roughly $e^{2k^{p/2}} \prod_{i=0}^{p-1} (k^i)!$ relative to q^r. We also propose two lower-bound conjectures and conjecture that our upper bounds are tight up to subexponential factors.

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